**Error in Numerical Approximation**

Error: Difference between exact and approximate solutions.

Types of errors:

1. Absolute error

→ |extact – approximate|

2. Relative error

→ absolute error / exact value or approximate value

3. Percentage error

→ relative error \* 100%

**Sources of Error:**

1. Round off error

= The error in the value which occurs due to rounding off a number to defines or desired decimal point.

2. Truncation error

= The error in the valve which occurs due to removing of parts of an extended expression. For example ex = 1+x/1! + x2/2! + x3/3! …. and only ex = 1+x/1! Is taken then truncation error occurs.

3. Inherent error

= The error which occurs due to mathematical assumption.

**Rules of rounding off a number:**

1. If (n+1)th digit is greater than 5, add 1 to nth digit.

2. If (n+1)th digit is less than 5, leave nth digit as it is.

3. If (n+1)th digit is exactly 5, add 1 to nth digit if there is at least one non-zero digit after 5.

4. If (n+1)th digit is exactly 5, add 1 to nth digit if there are no non-zero digit after 5 and a even number in nth digit, leave it as it is if there are no non-zero digit after 5 the nth digit is odd.

**Significant digit:**

Each of the digits of the numbers that are used to express it to the required degree of accuracy starting from non-zero digit.

**Rules of identify Significant digit:**

1. For decimal numbers, all digits from non-zero digits.

Example: 124 → all significant, 145.00 → all significant even the zeros after the decimal, 0.000123 → 3 significant digits

2. For non-decimal numbers, all digits from first non-zero digit to last non-zero digit.

Example: 123000 → 3 significant digits

**Upper limiting error of an approximate number:**

Exact value = x

Approximate value = xi

Absolute error = |x - xi|

Upper limiting error = |x – xi| < Δx

Example:

x = 1.231 xi = 1.23 Δx = 0.001

x = 1.232 xi = 1.23 Δx = 0.002 for all Δx ≤ 0.005

x = 1.233 xi = 1.23 Δx = 0.001

so, upper limitung error = ½ 10-N

**Maximum error:**

let u = f(x1,x2,…,xn) be multi-variable function.

Δxi be the error in xi then,

u + Δu = f(x1+Δx1,x2+Δx2,…,xn+Δxn)

= f(x1,x2,…,xn) + ((ẟf/ẟx1) Δx1+ (ẟf/ẟx2) Δx2 + … + (ẟf/ẟx3)Δxn) + 1/2!((ẟ2f/ẟx12) Δx12+ (ẟ2f/ẟx22) Δx22 + … + (ẟ2f/ẟxn2) Δxn2)

= u + ((ẟf/ẟx1) Δx1+ (ẟf/ẟx2) Δx2 + … + (ẟf/ẟx3)Δxn) + 0((Δx))2

error= Δu = n∑i=1 (ẟf/ẟxi) Δxi

Max.error = (Δu)max = n∑i=1 |ẟf/ẟxi| Δxi

**Problem:**

u = 10x2z2/y2 , x = 2.47 , y = 34.26 , z = 13.65. find the maximum relative error correct to 2 decimal points.

→ here,

Δx = ½\*10-2 = 0.005

Δy = Δz = 0.005

ẟu/ẟx = 20xz2/y2

ẟu/ẟy = 20x2z2/y

ẟu/ẟz = 20x2z/y2

At (x,y,z) = (2.47 , 34.26 , 13.65)

ẟu/ẟx = 7.84

ẟu/ẟy = 665.53

ẟu/ẟz = 1.42

Max.error = 7.84\*0.005 + 665.53\*0.005 + 1.42\*0.005

= 3.37

**Root finding Method:**

let f(x) = 0,

x = α is the root (solution)

if |xn -α|is sufficiently small,

x ≈ xn

**Rate of convergence:**

let there be x0,x1,x2,…. Be the approximations of the soln of f(x) = 0. Suppose there exists c>0 and ρ > 0 such that

limn→∞ |xn+1 -α|/|xn- α|ρ =c

then ρ is called rate of concergence

Ei = xi – α

limn→∞ |xn+1 -α|/|xn- α|ρ =c

limn→∞ |En+1 |/|En|ρ =c

limn→∞ |En+1 | =c|En|ρ (for sufficiently large n)

**Note:**

1. If ρ = 1 then method converges linearly

2. If ρ = 2 then method converges quadratically

3. If 1 < p < 2 then method converges superlinearly